Lloyd's Register
Yokohama DSO
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Nominal lateral load of the stiffener due to shear stress in 6/3.4.2.2 in CSR BC Rules

1. For the buckling of ordinary stiffeners, ultimate strength in lateral buckling mode in 6/3.4.2.2 in CSR BC Rules is to be complied with. In this requirement, the lateral buckling is treated as increase of the bending stress due to existence of the initial deformation under the following stress fields:

   - Compressive stress in stiffener direction,
   - Compressive stress vertical to stiffener direction,
   - Shear stress in the attached plate normal to the stiffener direction,

   In 6/3.4.2.2 of CSR BC Rules, these stresses are considered as a nominal lateral load for longitudinal stiffeners due to bending stresses (s_{xl} and s_y) and shear stress (τ_1), and is expressed as:

$$p_Z = \frac{t_a}{b}\left\{\sigma_{xl}\left(\frac{\pi b}{a}\right)^2 + 2c_y\sigma_y + \tau_1\sqrt{2}\right\} \quad (1)$$

   According to our study described in this document, however, it should be expressed as:

$$p_Z = \frac{t_a}{b}\left\{\sigma_{xl}\left(\frac{\pi b}{a}\right)^2 + 2c_y\sigma_y + \tau_1\right\} \quad (2)$$

   Hence, nominal lateral load due to shear stress should be modified with the reason in the sections hereafter. Considering the derivation of the following equation of bending moment due to the deflection *w* of stiffener in 6/3.4.2.2, the nominal lateral load can also be called 'pressure for unit deflection'.

$$M_0 = F_{Ki}\frac{P_z w}{c_f - P_z} \quad (3)$$

2. For pressure for unit deflection due to shear stress, the following assumptions are made in the Rules.

   a) When the shear stress τ is greater than the critical stress τ_{cr} of the plate panel of L x (2s), fitting of a stiffener is required to ensure its sufficient strength.

   b) The portion of the shear stress exceeding the critical value, i.e., $\Delta\tau = \tau - \tau_{cr}$, contributes to create the bending moment due to existence of the initial deflection.

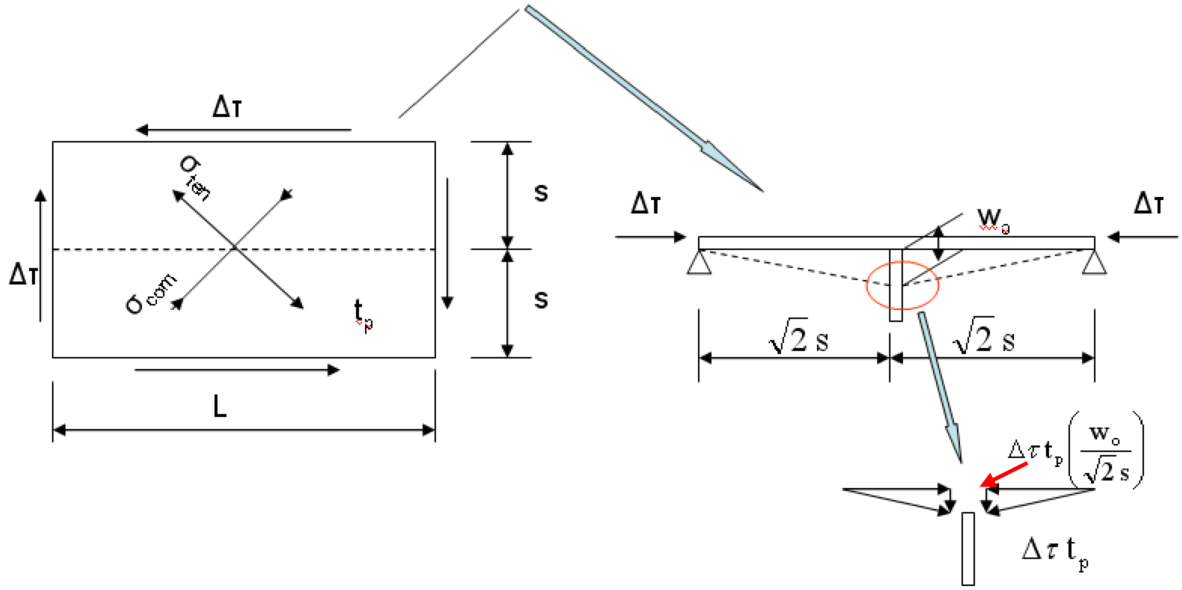

Fig.1

The shear stresses $\Delta\tau$ are expressed by the principal stresses σ_{ten} and σ_{com}, i.e.,

$$\Delta\tau = \frac{1}{2}(\sigma_{ten} - \sigma_{com}) \quad \text{and} \quad \sigma_{ten} = -\sigma_{com} \tag{4}$$

Or, the principal stresses are:

$$\sigma_{ten} = \Delta\tau \quad \text{and} \quad \sigma_{com} = -\Delta\tau \tag{5}$$

It is noted that the effect of tensile principal stress σ_{ten} is neglected in the Rules. Then a sinusoidal lateral load working downwards along the stiffener is obtained as follows:

$$q_S = 2\Delta\tau\, t_p \frac{w_0}{\sqrt{2}s}\left(\frac{1}{\sqrt{2}}\right) = \Delta\tau \frac{t_p}{s} w_0 \tag{6}$$

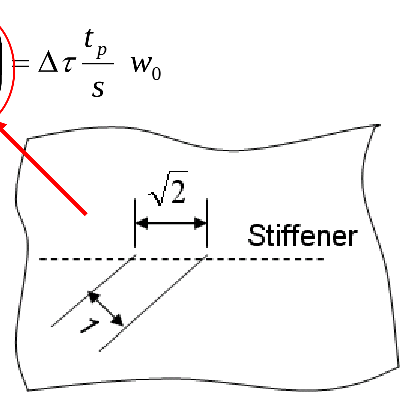

Fig.2

From the above, the pressure for unit deflection by the shear stress is obtained;

$$p_S = \frac{q_s}{w_0} = \Delta\tau \frac{t_p}{s} \tag{7}$$

S

On the other hand, the pressure for unit deflection by shear stress in CSR BC Rules (derived from eq.(1)) is;

$$p_Z = \tau_1 \sqrt{2} \frac{t_a}{b} \qquad (8)$$

Here it is noted that eq.(8) does not consider Fig.2 and it results in the difference between eq.(7) and (8).

3. Considering the above, it is concluded that nominal lateral load of the stiffener due to s_x , s_y and τ_1 for longitudinal stiffeners in 6/3.4.2.2 should be modified to:

$$p_Z = \frac{t_a}{b} \left\{ \sigma_{xl} \left(\frac{\pi b}{a} \right)^2 + 2 c_y \sigma_y + \tau_1 \right\} \qquad (9)$$

And that for transverse stiffeners also should be modified in the same manner.

.2 **Loads caused by compressive stresses vertical to stiffener direction**

The compressive stresses σ_y' acting vertically to stiffener direction are also causing vertical forces acting on the stiffener. This is shown in Figure 7 at half length of stiffener.

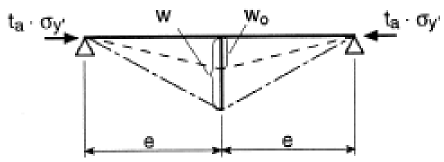

Fig. 7

The vertical force per unit length results from the geometry as follows:

$$p_{zq}' = 2 \cdot t_a \cdot \sigma_y' \cdot \frac{w_o}{e}$$

In order to proceed analogously to equation (11) similar to the local p_{zl}, which is related to the pre-deformation w_o, a local p_{zq} has to be found, which is related to the pre-deformation w_o. This load p_{zq} is:

$$p_{zq} = \frac{p_{zq}'}{w_o} = 2 \cdot \sigma_y' \frac{t_a}{e}. \qquad (12)$$

The factor 2 results from equal distances e on both sides of the stiffener. If the distances are not equal, a mean distance and a mean value for w_o may be taken.

If one substitutes e and ℓ by the corresponding dimensions a, b and n · b of the longitudinal and transverse stiffeners, one obtains the corresponding vertical forces, elastic support coefficients and bending moments.

.3 **Loads caused by shear stresses acting in the plate field's plane**

No simple, generally valid solution is available how to calculate the vertical forces caused by shear stresses. The difficulty lies in the selection of suitable pre-deformations, which correspond to the buckling deformations caused by shear stresses.

A procedure is, therefore, presented, which allows a reasonable estimate of the influence of shear stresses. This has been verified by several examples of typical hull structural elements. The formulae given in the GL-Rules have been derived as shown in Figure 8.

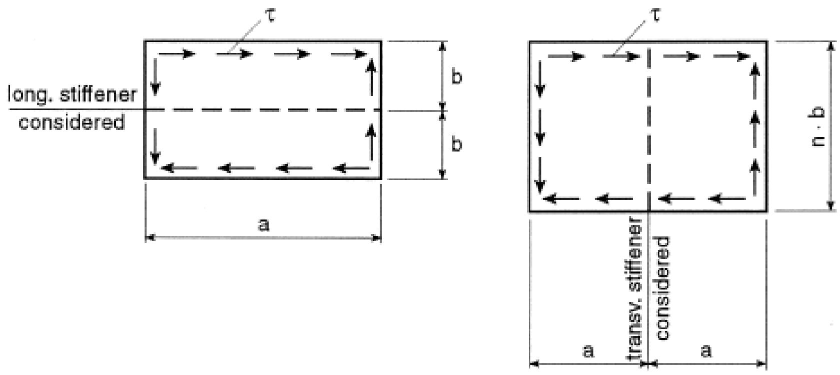

Fig. 8

The shear stress τ', which can be carried by the unstiffened equivalent single plate field may be calculated as for a single plate field of the size $n \cdot b \cdot i \cdot a$ and by using a safety factor S = 1 as follows:

$$\tau' = \frac{\kappa_\tau \cdot R_{eH}}{\sqrt{3}} = \frac{0{,}84 \cdot R_{eH}}{\lambda \cdot \sqrt{3}} \qquad (13)$$

where:

$$\lambda = \sqrt{\frac{R_{eH}}{K \cdot \sigma_e}} = \sqrt{\frac{R_{eH}}{K \cdot 0{,}9\,E\left(\frac{t}{n \cdot b}\right)^2}}$$

$$K = \sqrt{3} \cdot K_\tau = \sqrt{3}\left(4 + \frac{5{,}34}{\left(\frac{i \cdot a}{n \cdot b}\right)^2}\right) \quad \text{for} \quad \frac{i \cdot a}{n \cdot b} < 1{,}0$$

$$K = \sqrt{3} \cdot K_\tau = \sqrt{3}\left(5{,}34 + \frac{4}{\left(\frac{i \cdot a}{n \cdot b}\right)^2}\right) \quad \text{for} \quad \frac{i \cdot a}{n \cdot b} \geq 1{,}0$$

n = number of single plate field breadth

n = 2 for longitudinal stiffeners

i = 1 for longitudinal stiffeners

i = 2 for transverse stiffeners.

From equation (13) one obtains finally by insertion:

$$\tau' = t\sqrt{R_{eH} \cdot E\left(\frac{m_1}{a^2} + \frac{m_2}{b^2}\right)}. \qquad (14)$$

Where m_1 and m_2 are constants resulting from the aspect ratio of the equivalent plate field.

For $\tau < \tau'$ no stiffeners would be needed for S = 1,0. If $\tau \geq \tau'$, the equivalent plate field has no sufficient strength. Consequently the shear stress $\tau_1 = \tau - \tau'$ governs the design of the stiffener. For the calculation the vertical forces resulting from the shear stress, the latter is transformed in principal stresses as follows:

$$\sigma_{compression} = - \sigma_{tension} = \tau_1.$$

The tension and compressive stresses are acting vertically to each other (see Figure 9) and are turned by 45° against the stiffener axis.

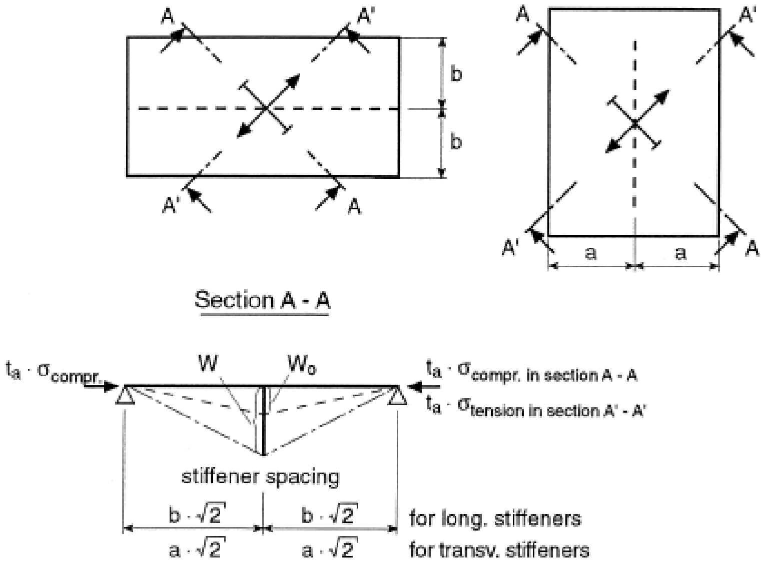

Fig. 9

If one neglects the supporting effect of tension stresses, one obtains analogously to the load caused by compressive stresses (see equation (12)) the load caused by shear stresses as follows:

$$p_{z\tau} = \sqrt{2} \cdot \tau_1 \frac{t_a}{b} \text{ for longitudinal stiffeners} \quad (15)$$

$$p_{z\tau} = \sqrt{2} \cdot \tau_1 \frac{t_a}{a} \text{ for transverse stiffeners} \quad (16)$$

The sum of loads caused by compressive and shear stresses is:

$$p_z = p_{z\ell} + p_{zq} + p_{z\tau}. \quad (17)$$